Geometric Sequences and Exponential Functions

Essential Question: How can you use geometric sequences and exponential functions to solve real-world problems?

Key Vocabulary
common ratio
(razón común)
explicit rule
(fórmula explícita)
exponential function
(función exponencial)
geometric sequence
(sucesión geométrica)
recursive rule
(fórmula recurrente)

KEY EXAMPLE (Lesson 15.1)

Find the common ratio r for the geometric sequence 2, 6, 18, 54, … and use r to find the next three terms.

$\frac{6}{2} = 3$, so the common ratio r is 3.

For this sequence, $f(1) = 2$, $f(2) = 6$, $f(3) = 18$, and $f(4) = 54$.

$f(4) = 54$, so $f(5) = 54(3) = 162$.

$f(5) = 162$, so $f(6) = 162(3) = 486$.

$f(6) = 486$, so $f(7) = 486(3) = 1458$.

The next three terms of the sequence are 162, 486, and 1458.

KEY EXAMPLE (Lesson 15.3)

Write an equation for the exponential function that includes the points $(2, 8)$ and $(3, 16)$.

Find b by dividing the function value of the second pair by the function value of the first: $b = \frac{16}{8} = 2$.

Evaluate the function for $x = 2$ and solve for a.

$f(x) = ab^x$ Write the general form.

$f(x) = a \cdot 2^x$ Substitute the value for b.

$8 = a \cdot 2^2$ Substitute a pair of input-output values.

$8 = a \cdot 4$ Simplify.

$a = 2$ Solve for a.

$f(x) = 2 \cdot 2^x$ Use a and b to write an equation for the function.

KEY EXAMPLE (Lesson 15.5)

Describe the transformations of the function $g(x) = 2(3)^x + 5$ as compared to the parent function $f(x) = 3^x$.

a has changed from 1 to 2.

This corresponds to a vertical stretch by a factor of 2.

The constant has changed from 0 to 5.

This corresponds to a translation of 5 units up.

$g(x)$ has been stretched by a factor of 2 and translated 5 units up.

EXERCISES

Find the common ratio r for each geometric sequence and use r to find the next three terms. *(Lesson 15.1)*

1. 1701, 567, 189,…

2. 5, 20, 80,…

Write a recursive rule and an explicit rule for each geometric sequence. *(Lesson 15.2)*

3. 4, 12, 36, 108, 324,…

4. 6, 30, 150, 750, 3750,…

Write an equation for the exponential function that includes the pair of given points. *(Lesson 15.3)*

5. $(2, 16)$ and $(3, 32)$

6. $(2, 4)$ and $(3, 2)$

7. Find a, b, and the y-intercept for $f(x) = 5(2)^x$, and then describe its end behavior. *(Lesson 15.4)*

MODULE PERFORMANCE TASK

What Does It Take to Go Viral?

You want your newest video to be so popular that it gets more than 750,000 daily views within a week after you post it. You share it with friends and assume that each friend will share the video with the same number of people that you do, and so on. How can you determine the smallest number of friends you need to show your video to? What answer do you think would be too big? Too small?

Start by listing how you plan to tackle the problem. Then complete the task. Be sure to write down all your data and assumptions. Then use numbers, tables, or algebra to explain how you reached your conclusion.

Ready to Go On?

15.1–15.5 Geometric Sequences and Exponential Functions

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Write a recursive rule and an explicit rule for each geometric sequence, and then find the next three terms. *(Lessons 15.1, 15.2)*

1. 2, 8, 32,…

2. 1024, 512, 256,…

Write an equation for the exponential function that includes the pair of given points. Find *a*, *b*, and the *y*-intercept, and then graph the function and describe its end behavior. *(Lessons 15.3, 15.4)*

3. $(1, 12)$ and $(-1, 0.75)$

4. $(-1, -8)$ and $(1, -2)$

5. Describe the transformations of the function $g(x) = 0.25(5)^x - 2$ as compared to the parent function $f(x) = 5^x$. *(Lesson 15.5)*

ESSENTIAL QUESTION

6. How does the rate of change of an exponential function behave as the value of x increases?

Assessment Readiness

1. Consider the geometric sequence 6, 24, 96, 384, …. Determine if each statement is True or False.

A. The sixth term is 1536.

B. The explicit rule is $f(n) = 6(4)^{n-1}$.

C. The recursive rule is $f(1) = 4$; $f(n) = f(n-1) \cdot 6$.

2. Tell whether the given system of equations has exactly one solution.

A. $\begin{cases} 3x - 2y = 6 \\ 2x + 2y = 14 \end{cases}$

B. $\begin{cases} y = -4x - 5 \\ y = -4x + 2 \end{cases}$

C. $\begin{cases} 5x - 3y = 15 \\ 5x + 3y = 15 \end{cases}$

3. Tell whether the given number is a term in both the sequence $f(n) = f(n-1) + 5$ and the sequence $f(n) = 3(2)^{n-1}$, if $f(1) = 3$.

A. 8

B. 18

C. 24

D. 48

4. A laser beam with an output of 6 milliwatts is focused at a series of mirrors. The laser beam loses 2% of its power every time it reflects off of a mirror. The power $p(n)$ is an exponential function of the number of reflections in the form of $p(n) = ab^n$. Write the equation $p(n)$ for this laser beam. Explain how you determined the values of a and b.